Programme of study: Mathematics (Key stage 4)

Curriculum aims

Learning and undertaking activities in mathematics contribute to achievement of the curriculum aims for all young people to become:

- successful learners who enjoy learning, make progress and achieve
- confident individuals who are able to live safe, healthy and fulfilling lives
- responsible citizens who make a positive contribution to society.

The importance of mathematics

Mathematical thinking is important for all members of a modern society as a habit of mind, for its use in the workplace, business and finance, and for both personal and public decision-making. Mathematics is fundamental to national prosperity in providing tools, for understanding science, engineering and technology, and for participation in the knowledge economy. The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised.

Mathematics equips students with uniquely powerful ways to describe, analyse and change the world. Students who are functional in mathematics and financially capable are able to think independently in applied and abstract ways, to reason, to solve problems and to assess risk.

Mathematics is a creative discipline. It can stimulate moments of pleasure and wonder for all students when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections.

Key concepts

There are a number of key concepts that underpin the study of mathematics. Students need to understand these concepts in order to deepen and broaden their knowledge, skills and understanding.

Competence in mathematical procedures

- Applying mathematical processes and algorithms accurately to a widening range of familiar and unfamiliar contexts within the classroom and beyond including managing money and other everyday uses of mathematics.
- Making choices about effective ways to communicate mathematical understanding.
- Using mathematical terminology and ideas accurately and coherently in spoken and written forms.
- Reading and understanding texts with mathematical content.

Creativity

- Making connections between different areas of mathematics and between mathematical techniques and problems or situations.
- Using existing mathematical knowledge to create solutions to unfamiliar problems.
- Posing questions and developing appropriate lines of enquiry.

Appreciation of mathematics

- Understanding that mathematics is both a tool for solving problems and a discipline with distinct structure.
- Gaining a sense of the history of mathematics and exploring how the mathematics of different cultures is present in modern mathematics.
- Being aware of some current applications of mathematics.
- Appreciating mathematics as an interesting and enjoyable activity in itself.

Critical understanding in using mathematics

- Recognising that a situation or problem can be represented using mathematics, that it can be represented in different ways and making connections between these representations.

Applying mathematical processes
For example, students could measure their height and weight, represent both quantities in decimal form, calculate their body mass index by substituting numbers into a formula and interpret the results or use statistical information to assess risk in everyday situations.

Algorithms
This includes knowledge and recall of number relationships and standard methods for adding, subtracting, multiplying and dividing.

Texts with mathematical content
For example, a newspaper, magazine or webpage including percentages or graphs, an atlas or a scientific text describing a relationship between variables.

Creativity
Students show creativity when problem-solving and problem-posing. They may approach tasks in unexpected ways using different mathematical techniques. Creativity can be encouraged by providing meaningful opportunities to experiment or to extend approaches to a problem.

Posing questions
The question that will unlock a problem might be the question 'what if…'

- a value or parameter is changed?
- an additional variable is introduced?
- a different approach altogether is used?

A tool for solving problems and a discipline with distinct structure
For example, mathematics can be used as a tool for making financial decisions in personal life and for solving problems in other fields such as building, plumbing, engineering or geography. Mathematics is a profession in its own right - professional mathematicians may work as statisticians or in operational research, for example.

History of mathematics
This includes understanding the motivation for the development of mathematics: knowledge of problems from the past that led to the development of particular areas of mathematics, an appreciation that pure mathematical findings sometimes precede practical applications and that mathematics continues to develop and evolve.

Mathematics of different cultures
For example, ancient and modern units of measurement, the contemporary use of Hindu-Arabic numerals and the derivation of the word 'algebra' from the name of a book by a Persian mathematician.

- Using mathematical ideas and models to explore real world issues and problems, recognising that solutions may need to take account of wider factors.
- Using deductive reasoning and proof as a tool for solving problems.
- Questioning, analysing and evaluating mathematical solutions.

Current applications of mathematics
This includes:

- examples of mathematical modelling in other disciplines including science and engineering
- mathematics within modern technology
- the role of probability in describing risk and uncertainty
- mathematical skills harnessed to ICT skills in the workplace.

A situation or problem can be represented using mathematics
This involves recognising types of situation or problem, acknowledging that not all situations can be represented mathematically, and making connections between the current situation and previous experiences.

Take account of wider factors
Mathematics equips students with the tools to model and understand the world around them. This enables them to engage with complex issues. For example, in financial capability mathematical skills are needed to compare different methods of borrowing and paying back, but the final decision may include other dimensions such as comparing the merits of using a credit card that promotes a particular charity with one offering the lowest overall cost.

Questioning, analysing and evaluating
It is important to be aware that mathematics can be used to inform and misinform.

Key processes

These are the essential skills and processes in mathematics that students need to learn to make progress.

Representing

Students should be able to:

- identify the mathematical aspects of the situation or problem
- compare and evaluate representations of a situation before making a choice
- simplify the situation or problem in order to represent it mathematically using appropriate variables, symbols and diagrams and models
- select mathematical information methods, tools and models to use.

Representing
Representing a situation places it into the mathematical form that will enable it to be worked on. It involves exploring mathematical situations independently; identifying the major mathematical features of a problem and potentially fruitful paths; using and amending representations in the light of experience; identifying what has been included and what omitted; breaking the problem down (for example, starting with a simple case, working systematically through cases, identifying different components that need to be brought together, identifying the stages in the solution process).

Identify
This includes identifying questions that can be addressed using statistical methods.

Representations of a situation
This includes moving between different representations in pure and applied contexts. For example in an engineering context or assembling a piece of flat-pack furniture.

Simplify
This involves using and constructing models with increasing sophistication and understanding the constraints that are being introduced.

Select mathematical information, methods, tools and models
This involves examining a situation systematically and identifying different ways of breaking a task down. It also involves identifying gaps in personal knowledge. In statistical investigations it includes planning to minimise sources of bias when conducting experiments and surveys and using a wide variety of methods for collecting primary and secondary data.

Analysing

Use mathematical reasoning

Students should be able to:

- make connections within mathematics
- use knowledge of related problems
- visualise and work with dynamic images
- look for and examine patterns and classify
- make and justify conjectures and generalisations, considering special cases and counter examples
- explore the effects of varying values and look for invariance
- take account of feedback and learn from mistakes
- work logically towards results and solutions, recognising the impact of constraints and assumptions
- identify a range of techniques that could be used to tackle a problem, appreciating that more than one approach may be necessary
- reason inductively and deduce.

Make connections
For example, realising that an equation, a table of values and a line on a graph can all represent the same thing or understanding that an intersection between two lines on a graph can represent the solution to a problem.

Use knowledge
This involves drawing on previous mathematical experience to select appropriate methods and representations.

Look for and examine patterns
This includes the use of ICT as appropriate.

Make and justify conjectures
These should be based on a secure grasp of the situation and previous experience.

Generalisations
Generalisations covering a range of mathematical content and contexts should be represented in different ways (including algebra).

Varying values
This involves identifying variables and controlling these to explore a situation. ICT could be used to explore many cases including statistical situations with underlying random or systematic variation.

Take account of feedback
This includes feedback that arises from implementing instructions using ICT.

A range of techniques
For example, working backwards, looking at simpler cases, choosing one or more of a numerical, analytical or graphical approach and being able to use techniques independently.

Tackle a problem
This includes using mathematical reasoning to explain and justify inferences when analysing data.

Reason inductively
This involves using particular examples to suggest a general statement.

Deduce
This involves using reasoned arguments to derive or draw a conclusion from something already known.

Use appropriate mathematical procedures

Students should be able to:

- make accurate mathematical diagrams, graphs and constructions on paper and on screen
- calculate accurately, using a calculator when appropriate
- manipulate numbers, algebraic expressions and equations and apply routine algorithms
- use accurate notation, including correct syntax when using ICT
- record methods, solutions and conclusions
- estimate, approximate and check working.

Interpreting and evaluating

Students should be able to:

- form convincing arguments to justify findings and general statements
- consider the assumptions made and the appropriateness and accuracy of results and conclusions
- appreciate the strength of empirical evidence and distinguish between evidence and proof
- look at data to find patterns and exceptions
- relate their findings to original question or conjecture, and indicate reliability
- make sense of someone else's findings and judge their value in the light of the evidence they present
- critically examine strategies adopted.

Communicating and reflecting

Students should be able to:

- use a range of forms to communicate findings to different audiences
- engage in mathematical discussion of results
- consider the elegance and efficiency of alternative solutions
- look for equivalence in relation to both the different approaches to the problem and different problems with similar structures
- give examples of similar contexts met previously and identify how they differed from or were similar to the current situation and how and why the same, or different, strategies were used.

Mathematical procedures
This includes procedures for collecting, processing and representing data.

Using a calculator when appropriate
For example, when calculation without a calculator will take an inappropriate amount of time.

Record methods
This involves increasing use of more formal methods, including algebra, and more formal proofs.

Interpreting
This includes interpreting data and involves looking at the results of an analysis and deciding how the results relate to the original problem.

Form convincing arguments
This involves using more formal arguments and proof to support cases and appreciating the difference between inductive and deductive arguments.

Strength of empirical evidence
This includes evidence gathered when using ICT to explore cases and understanding the effects of sample size when interpreting data.

Patterns and exceptions
This includes understanding that random processes are unpredictable.

Light of the evidence
Students may find, for example, errors in an argument or missing steps or exceptions to a given case. This includes interpreting information presented by the media and through advertising.

Critically examine strategies
This includes examining elegance of approach and the strength of evidence in their own, or other people's, arguments.

Communicating and reflecting
This involves communicating findings to others and reflecting on other approaches.

Range of forms
This includes appropriate language (both written and verbal forms), suitable graphs and diagrams, standard notation and labelling conventions and ICT models.

Alternative solutions
This includes multiple approaches and solutions using ICT.

Range and content

This section outlines the breadth of the subject on which teachers should draw when teaching the key concepts and key processes.

The study of mathematics should enable students to apply their knowledge, skills and understanding to relevant real-world situations.

The study of mathematics should include:

Number and algebra

- real numbers and their different representations
- rules of arithmetic applied to calculations and manipulations with real numbers including standard index form and surds
- proportional reasoning, direct and inverse proportion, proportional change and exponential growth
- upper and lower bounds
- linear and quadratic equations in one unknown
- simultaneous equations
- graphs of exponential and trigonometric functions
- transformation of functions
- graphs of simple loci

Geometry and measures

- properties of 2D and 3D shapes, and their applications including constructions, loci, geometric proof, Pythagoras' theorem, circle theorems and trigonometrical relationships
- properties and combinations of transformations including enlargements with negative scale factors
- 3D coordinate systems
- vectors in 2 dimensions
- conversions between measures and compound measures
- perimeters, areas, surface areas and volumes including those associated with parts of a circle

Relevant real-world situations
Mathematical skills are required in many workplace settings. For example understanding relationships between variables in stock control (food processing) or calculating and monitoring quantifiable variables of a hotel's performance (tourism).

Rules of arithmetic
This includes knowledge of operations and inverse operations and how calculators use precedence. For example, why different calculators may give a different answer for 1 + 2 x 3.

Calculations and manipulations with real numbers
This includes using mental and written methods to make sense of everyday situations such as temperature, altitude, financial statements and transactions.

Proportion
This includes percentages and applying concepts of ratio and proportion to contexts such as value for money, scales, plans and maps, cooking and statistical information (9 out 10 people prefer…).

Linear and quadratic equations
This includes relationships between solutions found using algebraic or graphical representations and trial and improvement methods.

Simultaneous equations
This includes one linear and one quadratic equation with whole-number solutions.

3D shapes
This includes cones and pyramids.

Applications
This includes making sense of plans, diagrams and construction kits.

Constructions, loci
This includes straight edge and compass constructions and constructions using ICT.

Geometric proof
This includes understanding and using congruence and mathematical similarity.

Transformations
This includes transformations using ICT and appreciating the use of symmetry in the built environment and art.

Areas, surface areas and volumes
This includes area of a triangle as ½ ab sin C, cones and spheres.

Statistics

- presentation and analysis of large sets of grouped and ungrouped data including box plots and histograms, lines of best fit and their interpretation
- measures of central tendency and spread
- experimental and theoretical probabilities of single and combined events
- applying statistics to enable comparisons and give evidence for associations and relationships.

Presentation and analysis
This includes the use of ICT.

Grouped and ungrouped data
This includes the use of two-way tables.

Measures of central tendency and spread
This includes using measures of average and range to compare distributions.

Probabilities
This includes applying ideas of probability and risk to gambling, safety issues and the financial services sector and simulations using ICT to represent a probability experiment, such as rolling two dice and adding the scores.

Combined events
This includes systematic approaches to listing all outcomes.

Applying statistics to enable comparisons
The statistics of distributions occur in many occupations for example in the health care, pharmaceutical and packaging sectors.

Curriculum opportunities

During the key stage students should be offered the following opportunities that are integral to their learning and enhance their engagement with the concepts processes and content of the subject.

The curriculum should provide opportunities for students to:

- work on sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts
- work on open and closed tasks in a variety of real and abstract contexts that allow students to select the mathematics to use
- work on problems that arise in other subjects and in contexts beyond the school
- work on tasks that bring together different aspects of mathematical content, involving use of several of the key processes, or require using the handling data cycle
- work collaboratively as well as independently to solve mathematical problems in a range of contexts, evaluating their own and others' work and responding constructively
- use a variety of resources when solving problems or carrying out mathematical procedures.

Other subjects
This includes geography, science, modern foreign languages, business subjects, design and technology, enterprise and economic well-being.

Contexts beyond the school
For example: conducting a survey into consumer habits; using formulas; planning a holiday budget; designing a product; and measuring for home improvements. Mathematical skills contribute to financial capability and to other aspects of preparation for adult life.

The handling data cycle
The handling data cycle is closely linked to the mathematical key processes and consists of:

- specifying the problem and planning (representing)
- collecting data (representing and analysing)
- processing and presenting the data (analysing)
- interpreting and discussing the results (interpreting and evaluating).

Work collaboratively
This includes talking about mathematics, problem solving in pairs or small groups and presenting ideas to a wider group.

Use a variety of resources
This includes using practical resources and ICT, such as spreadsheets and calculators, to develop mathematical ideas.