Programme of study: Mathematics (Key stage 3)

Curriculum aims

Learning and undertaking activities in mathematics contribute to achievement of the curriculum aims for all young people to become:

- successful learners who enjoy learning, make progress and achieve
- confident individuals who are able to live safe, healthy and fulfilling lives
- responsible citizens who make a positive contribution to society.

The importance of mathematics

Mathematical thinking is important for all members of a modern society as a habit of mind, for its use in the workplace, business and finance, and for both personal and public decision-making. Mathematics is fundamental to national prosperity in providing tools, for understanding of science, engineering and technology, and for participation in the knowledge economy. The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised.

Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world. Pupils who are functional in mathematics and financially capable are able to think independently in applied and abstract ways, to reason, solve problems and assess risk.

Mathematics is a creative discipline. It can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections.

Key concepts

There are a number of key concepts that underpin the study of mathematics. Pupils need to understand these concepts in order to deepen and broaden their knowledge, skills and understanding.

Competence in mathematical procedures

- Applying mathematical processes and algorithms accurately to a widening range of familiar and unfamiliar contexts within the classroom and beyond including managing money and other everyday uses of mathematics.
- Making choices about effective ways to communicate mathematical understanding.
- Using mathematical terminology and ideas accurately and coherently in spoken and written forms.
- Reading and understanding texts with mathematical content.

Creativity

- Making connections between different areas of mathematics and between mathematical techniques and problems or situations.
- Using existing mathematical knowledge to create solutions to unfamiliar problems.
- Posing questions and developing appropriate lines of enquiry.

Appreciation of mathematics

- Understanding that mathematics is both a tool for solving problems and a discipline with distinct structure.
- Gaining a sense of the history of mathematics and exploring how the mathematics of different cultures is present in modern mathematics.
- Being aware of some current applications of mathematics.
- Appreciating mathematics as an interesting and enjoyable activity in itself.

Critical understanding in using mathematics

- Recognising that a situation or problem can be represented using mathematics, that it can be represented in different ways and making connections between these representations.

Applying mathematical processes
For example, pupils could measure their height and weight, represent both quantities in decimal form, calculate their body mass index by substituting numbers into a formula and interpret the results, or use statistical information to assess risk in everyday situations.

Algorithms
This includes knowledge and recall of number relationships and standard methods for adding, subtracting, multiplying and dividing.

Texts with mathematical content
For example, a newspaper, magazine or webpage including percentages or graphs, an atlas or a scientific text describing a relationship between variables.

Creativity
Pupils show creativity when problem-solving and problem-posing. They may approach tasks in unexpected ways using different mathematical techniques. Creativity can be encouraged by providing meaningful opportunities to experiment or to extend approaches to a problem.

Posing questions
The question that will unlock a problem might be the question 'what if…'

- …a value or parameter is changed?
- …an additional variable is introduced?
- …a different approach altogether is used?

A tool for solving problems and a discipline with distinct structure
For example, mathematics can be used as a tool for making financial decisions in personal life, for solving problems in other fields such as building, plumbing, engineering or geography. Mathematics is a profession in its own right – professional mathematicians may work as statisticians or in operational research, for example.

History of mathematics
This includes understanding the motivation for the development of mathematics: knowledge of problems from the past that led to the development of particular areas of mathematics, an appreciation that pure mathematical findings sometimes precede practical applications and that mathematics continues to develop and evolve.

Mathematics of different cultures
For example, ancient and modern units of measurement, the contemporary use of Hindu-Arabic numerals and the derivation of the word 'algebra' from the name of a book by a Persian mathematician.

- Using mathematical ideas and models to explore real world issues and problems, recognising that solutions may need to take account of wider factors.
- Using deductive reasoning as a tool for solving problems.
- Questioning, analysing and evaluating mathematical solutions.

Current applications of mathematics
This includes:

- examples of mathematical modelling in other disciplines including science and engineering
- mathematics within modern technology
- the role of probability in describing risk and uncertainty
- mathematical skills harnessed to ICT skills in the workplace.

A situation or problem can be represented using mathematics
This involves recognising types of situation or problem, acknowledging that not all situations can be represented mathematically, and making connections between the current situation and previous experiences.

Take account of wider factors
Mathematics equips pupils with the tools to model and understand the world around them. This enables them to engage with complex issues. For example, in financial capability mathematical skills are needed to compare different methods of borrowing and paying back, but the final decision may include other dimensions such as comparing the merits of using a credit card that promotes a particular charity with one offering the lowest overall cost.

Questioning, analysing and evaluating
It is important to be aware that mathematics can be used to inform and misinform.

Key processes

These are the essential skills and processes in mathematics that pupils need to learn to make progress.

Representing

Pupils should be able to:

- identify the mathematical aspects of the situation or problem
- choose between representations
- simplify the situation or problem in order to represent it mathematically using appropriate variables, symbols, diagrams and models
- select mathematical information, methods and tools to use.

Analysing

Use mathematical reasoning

Pupils should be able to:

- make connections within mathematics
- use knowledge of related problems
- visualise and work with dynamic images
- look for and examine patterns and classify
- make and begin to justify conjectures and generalisations, considering special cases and counter examples
- explore the effects of varying values and look for invariance
- take account of feedback and learn from mistakes
- work logically towards results and solutions, recognising the impact of constraints and assumptions
- appreciate that there are a number of different techniques that can be used to analyse a situation
- reason inductively and deduce.

Representing
Representing a situation places it into the mathematical form that will enable it to be worked on. It includes beginning to explore mathematical situations, identifying the major mathematical features of a problem, trying things out and experimenting, and creating representations that contain the major features of the situation.

Identify
This includes identifying questions that can be addressed using statistical methods.

Simplify
This means appreciating that a model is a simplification of a situation.

Select mathematical information, methods and tools
This involves using systematic methods to explore a situation, beginning to identify ways in which it is possible to break a problem down into more manageable tasks, and identifying and using existing mathematical knowledge that might be needed. In statistical investigations it includes planning to minimise sources of bias when conducting experiments and surveys and using a variety of methods for collecting primary and secondary data.

Make connections
For example, realising that an equation, a table of values and a line on a graph can all represent the same thing or understanding that an intersection between two lines on a graph can represent the solution to a problem.

Use knowledge
This involves relating methods and representations to problems met previously.

Look for and examine patterns
This includes the use of ICT as appropriate.

Conjectures
This involves posing own questions.

Generalisations
This involves recognising the range of factors that affect a generalisation.

Varying values
This involves changing values to explore a situation, including the use of ICT. For example to explore statistical situations with underlying random or systematic variation.

Take account of feedback
This includes feedback that arises from implementing instructions using ICT.

Different techniques
For example, working backwards and looking at simpler cases.

Analyse a situation
This includes using mathematical reasoning to explain and justify inferences when analysing data.

Reason inductively
This involves using particular examples to suggest a general statement.

Deduce
This involves using reasoned arguments to derive or draw a conclusion from something already known.

Use appropriate mathematical procedures

Pupils should be able to:

- make accurate mathematical diagrams, graphs and constructions on paper and on screen
- calculate accurately, using a calculator when appropriate
- manipulate numbers, algebraic expressions and equations and apply routine algorithms
- use accurate notation, including correct syntax when using ICT
- record methods, solutions and conclusions
- estimate, approximate and check working.

Interpreting and evaluating

Pupils should be able to:

- form convincing arguments based on findings and make general statements
- consider the assumptions made and the appropriateness and accuracy of results and conclusions
- be aware of strength of empirical evidence and appreciate the difference between evidence and proof
- look at data to find patterns and exceptions
- relate findings to the original context, identifying whether they support or refute conjectures
- engage with someone else's mathematical reasoning in the context of a problem or particular situation
- consider whether alternative strategies may have helped or been better.

Communicating and reflecting

Pupils should be able to:

- communicate findings in a range of forms
- engage in mathematical discussion of results
- consider the elegance and efficiency of alternative solutions
- look for equivalence in relation to both the different approaches to the problem and different problems with similar structures
- make connections between the current situation and outcomes, and ones they have met before.

Mathematical procedures
This includes procedures for collecting, processing and representing data.

Using a calculator when appropriate
This means when the calculation is one the pupil currently cannot do without a calculator or when the calculation will take an inappropriate amount of time.

Record methods
This includes representing the results of analyses in several ways (for example tables, diagrams and symbolic representation).

Interpreting
This includes interpreting data and involves looking at the results of an analysis and deciding how the results relate to the original problem.

Evidence
This includes evidence gathered when using ICT to explore cases.

Patterns and exceptions
This includes recognising that random processes are unpredictable.

Someone else's mathematical reasoning
This includes interpreting information presented by the media and through advertising.

Communicating and reflecting
This involves communicating findings to others and reflecting on other approaches.

Range of forms
This includes appropriate language (both written and verbal forms), suitable graphs and diagrams, standard notation and labelling conventions and ICT models.

Alternative solutions
This includes solutions using ICT.

Range and content

This section outlines the breadth of the subject on which teachers should draw when teaching the key concepts and key processes.

The study of mathematics should enable pupils to apply their knowledge, skills and understanding to relevant real-world situations.

The study of mathematics should include:

Number and algebra

- rational numbers and their different representations
- rules of arithmetic applied to calculations and manipulations with rational numbers
- applications of ratio and proportion
- accuracy and rounding
- algebraic expressions, formulae, equations, inequalities and identities including index notation and the use of brackets to indicate precedence
- simultaneous linear equations in algebraic and graphical forms
- sequences, including those arising from rules, in a variety of contexts
- graphs of polynomial functions and their properties

Geometry and measures

- properties of 2D and 3D shapes and their applications, including constructions, loci and bearings, deductive reasoning and Pythagoras' theorem
- transformations, similarity and congruence including the use of scale
- points, lines and shapes in 2D coordinate systems
- units, compound measures and conversions
- perimeters, areas, surface areas and volumes

Rules of arithmetic
This includes knowledge of operations and inverse operations and how calculators use precedence. For example, why different calculators may give a different answer for 1 + 2 x 3.

Calculations and manipulations with rational numbers
This includes using mental and written methods to make sense of everyday situations such as temperature, altitude, financial statements and transactions.

Ratio and proportion
This includes percentages and applying concepts of ratio and proportion to contexts such as value for money, scales, plans and maps, cooking and statistical information (for example, 9 out of 10 people prefer…).

Accuracy and rounding
This is particularly important when using calculators and computers.

Algebraic expressions
This includes understanding that the transformation of algebraic expressions obeys and generalises the rules of arithmetic.

Equations
This includes setting up equations and analytical and numerical methods for solving them.

Simultaneous linear equations
This includes those with no solutions or an infinite number of solutions. Pupils should be able to recognise such special cases.

Properties
This includes gradient properties of parallel and perpendicular lines.

2D and 3D shapes
This includes circles and shapes made from cuboids.

Constructions, loci and bearings
This includes both straight edge and compass constructions and constructions using ICT.

Transformations
This includes appreciating the use of symmetry in art and transformations using ICT.

Scale
This includes making sense of plans, diagrams and construction kits.

Compound measures
This includes making sense of information involving compound measures, for example, fuel consumption, speed and acceleration.

Surface areas and volumes
This includes 3D shapes based on triangles and rectangles.

Statistics

- presentation and analysis of grouped and ungrouped data including time series and lines of best fit
- measures of central tendency and spread
- experimental and theoretical probabilities including those based on equally likely outcomes
- applying statistics to enable comparisons.

Presentation and analysis
This includes the use of ICT.

Spread
For example, the range.

Probabilities
This includes applying ideas of probability and risk to gambling, safety issues and simulations using ICT to represent a probability experiment, such as rolling two dice and adding the scores.

Statistics to enable comparisons
For example, using the shapes of distributions and measures of average and range.

Curriculum opportunities

During the key stage pupils should be offered the following opportunities, which are integral to their learning and enhance their engagement with the concepts, processes and content of the subject.

The curriculum should provide opportunities for pupils to:

- work on sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts
- work on open and closed tasks in a variety of real and abstract contexts that allow pupils to select the mathematics to use
- work on problems that arise in other subjects and in contexts beyond the school
- work on tasks that bring together different aspects of mathematical content, involving use of several of the key processes, or require using the handling data cycle
- work collaboratively as well as independently to solve mathematical problems in a range of contexts, evaluating their own and others' work and responding constructively
- use a variety of resources when solving problems or carrying out mathematical procedures.

Other subjects
This includes geography, science, modern foreign languages, business subjects, design and technology, enterprise and economic well-being.

Contexts beyond the school
For example: conducting a survey into consumer habits; planning a holiday budget; designing a product; and measuring for home improvements. Mathematical skills contribute to financial capability and to other aspects of preparation for adult life.

The handling data cycle
The handling data cycle is closely linked to the mathematical key processes and consists of:

- specifying the problem and planning (representing)
- collecting data (representing and analysing)
- processing and presenting the data (analysing)
- interpreting and discussing the results (interpreting and evaluating).

Work collaboratively
This includes talking about mathematics, problem solving in pairs or small groups and presenting ideas to a wider group.

Use a variety of resources
This includes using practical resources and ICT, such as spreadsheets and calculators, to develop mathematical ideas.